



# You Are Who You Eat With: Academic Peer Effects from School Lunch Lines

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#### Abstract

Using daily lunch transaction data from NYC public schools, I determine which students frequently stand next to one another in the lunch line. I use this 'revealed' friendship network to estimate academic peer effects in elementary school classrooms, improving on previous work by defining not only where social connections exist, but the relative strength of these connections. Equally weighting all peers in a reference group assumes that all peers are equally important and may bias estimates by underweighting important peers and overweighting unimportant peers. I find that students who eat together are important influencers of one another's academic performance, with stronger effects in math than in reading. Further exploration of the mechanisms supports my claim that these are friendship networks. I also compare the influence of friends from different periods in the school year and find that connections occurring around standardized testing dates are most influential on test scores.

JEL codes: C31, I21 Key words: Peer effect, network, education, lunch line

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### 1 Introduction

Schools are cliquish environments where students sort into social groups and develop friends, best friends, and acquaintances. The 2016 Gallup Student Poll finds that 84% of fifth graders have a best friend at school. It follows that within a classroom, students are not equally affected by each of their classmates – indeed, this would be a bold assumption. Yet little work examining peer effects attempts to measure variation in the strength of social ties. By misrepresenting the peer group in this way, we may bias estimates towards zero by overweighting unimportant peers and underweighting influential ones.

A growing body of research exploring peer effects point to the existence of peer effects in education. One important step towards harnessing the potential of peer effects is understanding their size. If we know peer effects exist but are negligible, it may not be worth attempting to harness their effects. But if they are large, it becomes deeply important to understand their impact on both treatments that explicitly rely on peer effects as a key mechanism (such as tracking, school choice, and school integration) and on treatments that do not (due to the multiplicative nature of spillovers). The ability to harness peer effects across all types of interventions could be quite powerful.

In this paper, I use daily lunch transaction data from NYC public schools to determine which students frequently stand next to one another in the lunch line as a 'revealed' friendship network. I use this friendship network to estimate academic peer effects in elementary school classrooms. School lunch is an unusual part of the day because student actions are not directly supervised and do not have direct academic consequences. It is a relatively unstructured and social time, during which students' primary concern is with whom they are able to socialize. This makes it an ideal context in which to observe social connections between students and the resulting peer effects.

I find significant peer effects for both math and reading test scores, with higher spillover effects in math than in reading. The network I construct is based on daily interactions, allowing me to explore the evolution of friendships over different periods in the school year. My results show that friends at the time of the test have the largest impact on test scores. This suggests that friendships change over the course of the school year, and the influence of those who are no longer friends may be short-lived relative to current friends.

The contributions of this paper are as follows. First, I measure opportunities for interaction between students in the lunch line and demonstrate a novel approach for revealing a friendship network. Data in which social connections are observed are rare and typically not administrative.<sup>1</sup> Second, my measure of social interaction allows me to both refine the classroom environment according to a revealed friendship network and properly weight classmates in order to overcome the binary nature of connections (students are either friends or they are not). I weight friendships on a continuous scale of importance, meaning that my results are not predicated on overweighting unimportant students and underweighting important peers as group averages necessarily do. Third, to my knowledge this paper presents an empirical application with the largest set of partially overlapping networks (one for each day of the school year) such that the measure of connection between individuals can be thought of as continuous. Finally, each student has a unique reference group allowing me to decompose the peer effect into its social and contextual components,<sup>2</sup> a distinction which is important for accurate estimation of spillover effects.

This paper continues as follows. In Section 2, I motivates this work in the context of related research. I then describe the data I use in this paper and how I construct the sample in Section 3. Section 4 discusses how I measure connection between students. Once we understand the network structure, I discuss identification issues, how I overcome them, and the model I estimate in Section 5. Section 6 report my baseline results and shows that a student's friends have a significant and sizable influence on their academic performance. I

<sup>1.</sup> Friendship surveys may be unreliable or include significant noise. Landini et al. (2016) finds discrepancies between parent and student friendship surveys, and highlights the lack of reciprocity in both surveys. Higher reciprocity leads them to treat the student survey as correct, but the large number of unreciprocated connections may be concerning - both in the survey used in this paper and in work using Add Health or other friendship surveys.

<sup>2.</sup> These effects are also called the endogenous and exogenous effect, as in Manski (1993). More discussion of the identification of these effects can be found in section 5.1.

conduct a number of robustness checks in Section 7. This includes providing evidence that the network is indeed a friendship network as well as exploring the effects of friends at different points in the school year. Section 8 concludes.

# 2 Background

Social networks are by their nature hierarchical and complex, and each individual is uniquely impacted by a different set of peers. Understanding which peers are relevant is critical to identifying meaningful estimates of peer effects - both for research relevance and tautologically as many models include average group characteristics or outcomes. A researcher decides which peers are relevant for each student and the structure of that network. This is known as the student's reference group. If the reference group has little impact on a student, or is too broadly defined, we may understate the importance of peers. However if we define the set of peers too specifically, we may miss other influencers and misstate their importance.

A primary focus of the education peer effects literature is the effect peers have on test scores. It is important to note that estimates vary widely<sup>3</sup> due to difference in context, methodology, and reference group definition. The discussion that follows focuses on reference group definitions used in the literature, but Vigdor and Ludwig (2010), Epple and Romano (2011), Sacerdote (2011), and Paloyo (2020) give excellent reviews of the academic peer effects literature broadly.

We see notable variation in who researchers include in the reference group. Researchers frequently use cohort (school-grade level) as a reference group in order to avoid concerns around within-school sorting into classrooms (Hoxby 2000, Vigdor and Nechyba 2007, and Carrell and Hoekstra 2010). Burke and Sass (2013) find cohort effects near zero using a student fixed effect model incorporating average peer performance. However, they find that

<sup>3.</sup> In Sacerdote (2011), the results in Table 4.2 illusutrate the wide range of estimates we see in the literature: from slight negative in Vigdor and Nechyba (2007) near -0.1 to slight positives in Burke and Sass (2007) near 0.05 to Hoxby (2000) with large estimates of 0.3 to over 6. The papers I highlight here show the range of estimates, but most in the literature fall near the middle range.

classroom peers are more important than cohort peers and produce meaningful peer effects. Their results suggest that while choosing the cohort is a convenient way to avoid selection problems, estimates derived from this reference group may understate the overall peer effect. This is intuitive, as students in other classrooms generally have fewer opportunities to interact and influence behavior than classmates do. Incorporating unimportant peers who contribute little to the outcome should bias estimates towards zero. Thus for questions of whether peer effects exist, detecting peer effects at the cohort level may be sufficient. But it becomes important to model the network as precisely as possible in order to measure the size of peer effects.

It is sometimes possible to zoom in further than the classroom level to examine the effect of students who almost certainly are in one anothers' social network. Examples include assigned seat neighbors (Hong and Lee 2017) and college roommates (Sacerdote 2001, Zimmerman 2003, and Stinebrickner and Stinebrickner 2006). Notice that even as these reference group definitions may better capture members of the peer group, connections are still binary.<sup>4</sup> There is a tension here, which is that as the reference group narrows and the target group is more likely to be peers, more actual peers are almost certainly missed as well. Both of these issues are mediated under a scenario in which peers are revealed by student behavior and when the measure of connection is continuous.

Other work also explores the effect of homophily on peer effects by breaking the environment into groups based on shared characteristics. Arcidiacono and Nicholson (2005) study peer effects in medical school cohorts. They break up the cohort and investigate heterogeneous effects based on shared race and shared gender, finding effects along gender lines but not along racial ones. Lavy et al. (2012) finds girls are affected by high performing peers, while boys are not. Similarly, Fletcher et al. (2020) explores within-gender friendships and finds girls but not boys perform better when friends with peers who have college educated

<sup>4.</sup> It is important to note that there is reason not to believe that roommates are likely to be meaningful members of a student's peer group. Stinebrickner and Stinebrickner (2006) has a nice discussion of these and related potential concerns.

mothers.

The work discussed so far focuses on how researchers have defined the peer environment (ex: classroom or cohort), and how that environment can be sliced to focus on relevant subgroups. Another way to define the reference group is to look beyond the school, cohort, or classroom entirely. This is possible when information on the network structure itself is observed. The National Longitudinal Study of Adolescent to Adult Health (Add Health) is a workhorse in the peer effects literature as a result of its friendship survey of middle and high school students.<sup>5</sup> The friendship survey allows researchers to define a set of relevant peers from the school (networks are within school, rather than within classroom), but the researcher does not know the relative importance of these students (ex: who is the student's closest and most influential friend).<sup>6</sup>

Most work focuses on the scope of the reference group - defining (or assuming) who is and who is not a relevant peer. Little work measures connection strength - how influential each relevant peer is. Lin (2010) points out that "the ideal model should contain the weighted average of the peer variables, with weight determined by the importance of a friend, as opposed to a mean peer variable" before noting that this sort of data is not common.

In this paper, I define the scope of the reference group as the classroom. I then measure the intensity of connection between classmates and use this network structure to determine social spillovers on math and reading test scores. In order to measure connection intensity, I use administrative point of service data from the New York City Department of Education to observe daily lunch transactions. I use these daily observations to determine which stu-

<sup>5.</sup> A litany of papers have been written using Add Health, including Lin (2010), Bifulco et al. (2011), Lin and Weinberg (2014), Hsieh and Lin (2017), and Patacchini et al. (2017). The appeal of this data, which surveys and follows up with students who were 7-12th graders in US public schools during the 1994-1995 school year, is that it asks students who their friends are and includes survey responses on a variety of outcomes from GPA to smoking and drinking habits.

<sup>6.</sup> There are exceptions. Patacchini et al. (2017) divides the networks into students who are friends in both waves and those who are not, finding that students who are long-term friends are more important than those who are friends in just one, both in the short and long term. Hsieh and Lin (2017) explores connection variation based on gender and racial homophily. Lin and Weinberg (2014) explores differences in effect based on whether the friendship was reciprocated or not. In each of these, connections are categorized differently (ex: a separate network is used for long and short term peers in Patacchini et al. (2017)), but remain binary.

dents frequently stand near one another in the lunch line as a measure of friendship. This friendship network has several important features. First, the measure of contact is based on administrative data rather than surveys.<sup>7</sup> This means that friendships are revealed rather than stated, and their revelation means that the friendships need to be reciprocated.<sup>8</sup> Additionally, friendships change during the school year, and we observe the result of daily decisions students make rather than a single survey snapshot. Second, our measure of contact between students occurs during lunch, which is an important social space. Lunch is a relatively unstructured environment within school, allowing students more freedom to interact outside direct supervision from teachers and without direct academic consequences. This makes connections observed during the lunch period socially meaningful. Third, connections can have varying strengths,<sup>9</sup> and I allow them to vary on a continuous scale of importance. Next, these networks are constructed fresh during the school year as students do not have control over their classroom assignment, and we can observe how friendship significance evolves over time. Finally, because this network is individual-specific, we do not observe the perfect collinearity between group mean characteristics and mean expected group outcome (the reflection problem).<sup>10</sup> This enables us to disentangle the peer effect (Manski 1993 calls this the endogenous effect) from the contextual effect (sometimes called the exogenous effect). This is a policy relevant distinction, as the peer effect is a multiplier and the contextual effect is not.<sup>11</sup>

<sup>7.</sup> There is some concern that friendship surveys may not be accurate. They could be aspirational (which could explain unreciprocated friendships in Lin and Weinberg 2014). Landini et al. (2016) provides evidence of unreliability in friendship surveys by comparing a student friendship survey to a survey of who their parents think are their friends. Both display a significant number of unreciprocated friendships (as in Add Health), although the parent survey displays more.

<sup>8.</sup> Lin and Weinberg (2014) use Add Health to show that reciprocated friendships are stronger than unreciprocated friendships on a variety of outcomes, including academics.

<sup>9.</sup> For example: student A considers student B to be her best friend, student C is a friend, and student D is simply a classmate.

<sup>10.</sup> As described in Manski (1993), and in section 5.1.

<sup>11.</sup> For example, Hoxby (2000) shows that elementary students benefit from the presence of a higher percentage of girls in the elementary school classroom. What this does not tell us is whether interaction with girls is important (the peer effect), or if it is related to, for example, behavioral differences where young boys demand more teacher time (an contextual effect). If the former is true, it may be desirable for classrooms to be structured such that student interaction in mixed gender groups is increased. If it is the latter it may be beneficial for teachers to have an aide help manage student behavior.

## 3 Data and Sample Construction

#### 3.1 Data

This paper uses student-level administrative data on student academic performance, school and classroom codes, and socio-demographic characteristics from the 2018 academic year, with lag outcomes coming from the 2017 academic year. I link this to student lunch transactions at the point of sale (POS) for 2018.

Student-level demographic data comes from the New York City Department of Education (NYCDOE). These data include socio-demographic characteristics such as gender, race, age, grade, residential zip code, an indicator of eligibility for free or reduced-price lunch, and whether a student is an English language learner. Data also include class assignment as well as both current year and lagged mathematics and reading test scores.

POS data indicate the exact timing of lunch purchase transactions for students (precise to the second) for every day during the school year. I use this transaction information to observe the order of students in the lunch line and measure social connections in the classroom by observing which students are often in proximity to one another in the lunch line.

#### 3.2 The point of sale system

NYCDOE began implementing a POS system in their school cafeterias in 2010. By academic year 2018, 88.0% of schools had the system installed at the start of the school year. These schools served 90.2% percent of the over one million students in the school district. Implementation started in large schools first, with a focus on middle and high schools where the district felt these systems would do the most good. However, by academic year 2018 the system was in 93.4% of elementary schools, and these schools served 95.5% of grade 1-5 students.

Table 1 shows that the makeup of the schools with POS systems is slightly different than the full school district. Students in schools with a POS system are more likely to be Hispanic or Asian/other and less likely to be black. This is likely because schools with POS systems are more likely to be located in Staten Island and Queens, and less likely to be in the Bronx or Brooklyn. These differences, while statistically significant, are not large.

The primary way students interact with the POS system is either by entering a PIN in a keypad or a cafeteria worker uses a list of names and faces to enter the transaction as students move through the line. This is not standardized over the district, can vary by school, and is not observed.

#### 3.3 Sample

The sample is taken from the universe of students in the NYC public schools for academic year 2018. I examine elementary school students for two main reasons. First, elementary school students typically remain with the same set of classmates for the school day, as opposed to middle and high school students who tend to switch classes. Second, elementary school students are more likely to participate in the school lunch program than middle or high school students. This is likely because they have less autonomy and do not have the same outside options as older students who may be allowed to go off campus during lunch. School lunch participation for our sample is 65.8%. Additionally, I limit analysis to students in general education classrooms.

Table 2 illustrates the sample selection process. I begin with all general education fourth and fifth grade students in schools with a POS system in place for the entire academic year.<sup>12</sup> I cannot measure social connections to students who never participate in school lunch, and 3.7% of students fall into this category. I lose 7.5% of students because they are missing either a current year score or a lag test score for both math and reading. Some students do not participate in standardized tests, so I lose another 2.3% of students from test nonparticipation in both math and reading. I drop less than half a percent of students total due to the following three reasons. First, I exclude lunch transactions occurring before 10am and after 2pm. Transactions occurring outside this window are rare and may be improperly coded

<sup>12.</sup> I restrict to fourth and fifth grade students because standardized tests begin in the third grade, and I include a lag test score in the model.

breakfast transactions, transactions entered after the fact (such that timing is not indicative of the lunch line order), or simply an unreasonable assigned lunch time.<sup>13</sup> Second, I remove transactions occurring more than an hour earlier or later than the mean transaction time for a classroom. These students appear to be "out of line", and as a result are not relevant for determining who is next to whom in the lunch line. Including them would simply add noise to the estimates, so I remove these transactions. Third, I remove transactions which occur simultaneously for the entire classroom. This is indicative of an unusual event, such as a field trip, and gives no information relevant to the lunch line order. The final exclusion I make is students in classrooms with less than 20 students, resulting in the largest loss of students from the sample (15.78%). I choose to look only at classrooms that are larger than twenty students because I am concerned classrooms that appear smaller may be integrated co-teaching (ICT) classrooms, and I do not want unobserved peers.<sup>14</sup>

Table 3 reports some summary statistics regarding this sample . The sample includes fewer black students and more Hispanic and Asian/other students. This is likely the result of where the POS systems have been implemented, as the Bronx and Brooklyn are underrepresented while Queens and Staten Island are overrepresented in locations having received POS systems. Because implementation has occurred in a large proportion of schools, discrepancies are small. Test scores are normalized z scores across grade level in the school district, so the sample is slightly higher performing than average. Average class size in my sample is 25.6 students, and the lunch participation rate is 65.8%. Math and Reading scores are z-scores standardized to zero for the entire NYC public school student population.

<sup>13.</sup> Some lunch times are even more unreasonable than these bounds we place on lunch times, as in the article Brand (2019) "Why do some NYC school kids still eat lunch before some of us have had breakfast?" However, times like these are even more of an anomaly for elementary students than the high schoolers discussed in the article.

<sup>14.</sup> ICT classrooms combine general education students and students with disabilities together. Students learn from the general education curriculum and are taught by a team of two teachers: one general education teacher and one special education teacher. ICT classrooms typically have a ratio of 40% students with disabilities and 60% general education students.

### 4 Network Construction

### 4.1 Defining Social Distance

This paper uses a novel approach to measure contact between students and reveal the classroom friendship network. I observe the timing of every lunch transaction in the POS system every day during the school year, and this timing is precise to the second. This allows me to observe the lunch line both in terms of physical order and in the timing of movement through the line. I use this information to construct a peer network, but first discuss how to extract a meaningful social distance from this information.

I transform the near-continuous timing data for each day into ordinal data. This allows us to think about distance as the physical proximity of students to one another. Because lunch is a relatively unstructured and social time, a primary concern for students is who they are able to socialize with in the line and then during lunch. The simplest way to transform the observed order into a social distance is to look at whether any two students i and j are within some threshold distance (number of students) of one another. My baseline model uses a threshold distance of one - whether two students are next to one another in line. For robustness, I also look at larger threshold distances in Section 7.2.

It is worth discussing the implication of the observed lunch line order, as the ordering process is a black box, and the method of ordering likely varies by classroom. I discuss some possibilities for how students are ordered, fitting them into three categories: students have agency over their choice of line position, students are ordered by someone else (such as the teacher), and students have agency within a constraint. I then provide evidence that in the majority of classrooms, students either have at least some agency over their position in line.

First I discuss situations in which students have agency over their position in line. Students must balance a choice between being in line with their friends and their preference for being towards the front or back of the line. For most students, I believe the choice of being in line with friends is more important than their line position. If this is true, then it is clear that the line order contains information relevant to the social network in the classroom. However, it is possible that many students' preference for being at the front of the line (for example) dominates their desire to be near friends. A classroom in which all students wish to be first would see a race to the front of the line. Thus line order depends upon classroom geography and where students sit in relation to the door (start of the line), with students sitting near one another tending to line up near one another. If students sitting near one another are more likely to talk to one another or work together during class time, then this gives us another reason physical proximity in the lunch line would be socially important. In both of these situations, students who are near one another in the lunch line would be expected to be more influential in one another's social network - at least as it relates to academics within the classroom - than a randomly selected classmate. The truth is likely some combination of these two situations. For students geographically near the door, they have the option to be first or wait for their friends. Those further from the door do not have this choice. Thus in a classroom in which all students wish to be first, the benefits to rushing decrease in distance from the door. A tipping point could occur at which point students switch from racing to the front of the line to waiting for their friends.

Second, it is possible that students could be ordered by their teacher according to some metric - perhaps alphabetical. We do not observe names, and so we cannot test this hypothesis directly (although we do look at how much strict ordering exists in our sample). If students are ordered based on name, we expect little reason for these students to be socially more important than other students.<sup>15</sup> The teacher could order students by some other method perhaps according to student characteristics (demographic, performance, or behavioral). If we believe that students with similar characteristics are more likely to be friends with one another (homophily), then observing similar characteristics in students near one another may be indistinguishable from an external ordering placed upon the students according to this

<sup>15.</sup> Outside the notion that students of similar cultural or ethnic backgrounds might have similar names and thereby be grouped together. While some work looks at the ability to predict ethnicity based on names, such as Elliott et al. (2009) and Ryan et al. (2012), the success of these algorithms is still limited. Predicting ethnicity based on alphabetical ranking within an average group size of around 25.6 would be unsuccessful.

same set of criteria. These students may also be more socially important to one another than a randomly selected classmate, as Horrace et al. (2020) shows.

Finally, there is the possibility that students sort into the lunch line based on some combination of autonomy and rules. For example, the teacher may dismiss students by classroom tables, so that students form a line within a subset of the classroom - they have autonomy within a constraint.<sup>16</sup> Students face a similar decision whether to line up next to friends (within the constraint group) or in terms of optimal position. Notice that both physical and social positioning are constrained, as a student with preference for the front of the line may not have a choice over line position until the first half of the line is filled. Similar to when students have full autonomy, line order likely reflects some level of student importance - either through selecting friend groups or the importance of the constraint group (such as classroom geography). The result is similar to that of full autonomy, but the effect of these peers is likely smaller than under full autonomy, as this is a group of "next-best" friends.

While the line-up process is itself unobserved, I provide evidence that students have at least some agency over this decision by considering whether students are ordered into roughly the same order each day. To do this, I construct a measure of within-classroom noise as detailed in Appendix B. The measure M is based on the number of order inversions (swaps in the order of students i and j) observed in the order over the year, and it is normalized such that it is invariant to classroom size and participation rate. Figure 2 shows the distribution of that measure and that the bulk of classrooms (average measure value is 0.203) are closer to a uniformly random distribution (value of 0.25) than fully ordered (value of zero), but that there is more order than complete randomness.<sup>17</sup> This is consistent with the idea that students in most classrooms have agency over their position in line, and choose positions in ways that are varied but less than random (ex: in order to be with their friends). It is

<sup>16.</sup> We may think this might elicit a similar peer effect to that in ??, in which they measure the peer effect for Korean college students sitting next to each other in classrooms with assigned seats. However, I observe most students near more than just a handful of their classmates in the lunch line.

<sup>17.</sup> Appendix B outlines how the measure behaves under changes in class size, participation rate, and levels of randomness.

important to note that the distribution of this measure has a small tail with what may be considered abnormally low noise (where we may think classrooms are ordered). If we let 0.1 be the threshold below which classrooms are ordered, about 3% of classrooms are ordered.<sup>18</sup> In Section 7.3 I remove these classrooms as a robustness check and test other thresholds.

### 4.2 Scaling from daily observations to the friendship network

I observe daily lunch transaction timings over the entire school year, which I translate into the lunch line order for each day. The next step is to zoom out to the full year, such that students observed in frequent close proximity to one another on individual days are considered friends. Some students do not participate in school lunch every day (or are absent from school), complicating this process. On a day that a student does not attend school, we miss their signal of who they would choose to stand in line next to on that day, and they also limit the choice set of the students who remain (by removing themselves from the candidate pool).

I start constructing the network by averaging daily observations together, akin to what De Giorgi et al. (2010) do with classes.<sup>19</sup> This proximity matrix represents the percent of days each student is near each other student. We may be concerned that students with low participation will appear to have artificially low connections measured by this proximity matrix. There are two ways to address this. First, I can simply row-normalize the average proximity matrix such that each row sums to one. Row normalization is common in the literature because it improves interpretability of results by appropriately weighting influential peers for the given student such that we have the weighted average (characteristics or outcome) of the reference group. Each row i indicates student i's relevant peers, appropriately weighted. I row normalize for the interpretation benefits, but it is important to notice that row-normalization

<sup>18. 134</sup> of 4,077 classrooms are below the 0.1 threshold.

<sup>19.</sup> De Giorgi et al. (2010) provides an empirical example of partially overlapping networks. Students are randomly assigned to nine college courses. Strength of connection between individuals is based on the number of courses students take with one another, and the authors estimate an overall peer effect for this network. We can think of the networks in this paper as a large number of partially overlapping networks (for each day of the school year). To my knowledge, this is the largest set of partially overlapping networks used, and the result is a network of connections that are essentially continuous.

changes the interpretation of the proximity matrix from the percent of school days both students are near one another to be the percent of days student i is present that student i was near every other student j. For student i with low participation, this moves their average connections with students from near zero to the percentage of times i participated and was near each other student j. By increasing the weight on the days a student does participate, I have addressed the issue of not observing who a student would choose to be near if they did attend. However, it is not clear that we have addressed the second problem in which student i is removed from the choice set of other students.

In order to address this second concern, I construct a proximity matrix for the percent of times we observe students near one another when both are present:

$$p_{ij} = \frac{\sum_{d=1}^{D} S_d(i,j)}{\sum_{d=1}^{D} \delta_d(i,j)} \text{ for } i \neq j \text{ ; and } p_{ij} = 0 \text{ for } i = j$$
(4.1)

where  $S_d(i, j)$  indicates that students *i* and *j* are next to one another on day *d* and  $\delta_d(i, j)$ indicates that both *i* and *j* are participating in lunch on day *d*. The proximity matrix is then  $P = \{p_{ij}\}$ . As mentioned, for estimation we create our weighting matrix *W* by rownormalizing the proximity matrix such that each row sums to one. While averaging the daily observations as done in De Giorgi et al. (2010) and the method described in equation 4.1 lead to different proximity matrices, row normalization makes the resulting weighting matrices identical. Figure 1 provides a simple example to illustrate how I convert daily observations into a proximity matrix (according to equation 4.1) and corresponding weighting matrix. In the example, we observe five students over six days, and one student is absent or not participating each day. Figure 1a shows the daily proximity matrix for each individual, where a dark square illustrates connection and a white square represents no connection. Notice that students who are at the front or the back of the line have only one connection, while every other student has two. Part 1b applies equation 4.1 to the daily observations, calculating the percent of days both students are present for which they are next to one another. We now have a continuum of connection strengths and the darker the square the stronger the connection. Figure 1c shows the result of row normalization. Graphically, it appears that row normalization has dampened the effect, but this is not the case. Instead, it proportionally reweights the proximity matrix so that each row, when multiplied by Y or X, creates a weighted average of the relevant peer outcome or characteristics.

Admittedly there are other ways we could construct the proximity matrix, and there are potential concerns with the way we have constructed ours. Perhaps most concerning is that low-participation students could appear overly important for those they stand near when they do participate. I address this using an alternate proximity matrix in Section 7.

It is also important to distinguish between absence and non-participation. The previous discussion dealt with absence from school, but non-participation adds an additional complication. The majority of non-participation in elementary school lunch is because students brought their own lunch from home. Thus if a student is present at school, but not participating in lunch, they are likely present in the lunch line - at least during travel from the classroom to the cafeteria - and importantly they are part of the decision process when students decide where to stand in line. Thus two students we observe as being next to one another may actually have another student between them (or more than one) during the decision process for who to stand near. This is an issue of truncated data, and likely a significant source of noise in the model. The result is that an observed distance of one between two students is actually a distance of at least one. This means connections we observe are weaker than actual connections in the classroom, and results obtained from this data are likely a lower bound on the peer effect from lunch-mates.

### 5 Methods

### 5.1 Identification

Identification of peer effects is notoriously difficult due to the reflection problem and selection into peer groups. In this section, I discuss these threats and how I address them in this paper.

In his seminal paper, Manski (1993) discusses three different effects which may be captured in a naive model of peer effects and relates these effects to the reflection problem. The first is the social effect,<sup>20</sup> which is often the effect of interest to researchers and policymakers. The social effect results when one student's performance affects the performance of another. For example, we observe a social effect when for two students working together on a group project if the performance of one student varies based on skill level (performance) of that student's partner. This is of interest to policymakers because the social effect is a multiplier, capturing spillovers to other students due to interaction. The second effect is the contextual effect, sometimes called the exogenous effect. Contextual effects control for student characteristics in the reference group. For example, we might expect that wealthier students perform better on tests, all else equal. As a result classroom performance may increase with wealth, but this is due to student characteristics rather than student interactions. The final effect Manski (1993) discusses is the correlated effect. This is often not a social effect at all, but is related to common exposure by students to the same treatment. For example, a lack of adequate facilities or a good teacher are felt by all students in the classroom, but these factors are not related to the students or any social interactions between them.

In many reduced form models of peer effects, we cannot distinguish between contextual and endogenous effects because the performance of the reference group is collinear with the characteristics of this group. This is known as the reflection problem (Manski 1993). However, when individuals have unique reference groups, this is sufficient to separately identify social and contextual effects (Bramoullé et al. 2009). This is because the collinearity issue arises when individuals share a reference group. In this paper, individual level reference groups arise because each student is next to a unique set of students each day (another student will likely be next to one of the students, but there cannot be more than one student next to

<sup>20.</sup> Manski 1993 calls this the endogenous effect. The endogenous effect is so named because it directly places the outcome vector on the right hand side of our model. Use of the linear in means model structure (assuming the peer effect is a weighted average of peer performance) and maximum likelihood estimation allows us to solve for this endogeneity in our results. More details about the model follow in Section 5.2 and about the estimation procedure in Appendix A. This paper uses the term social effect but these terms are equivalent.

both students). We may be concerned that averaging over the days could cause different students to have the same reference group. However, with 180 days and differing levels of participation among students this does not occur in my sample. Using a network composed of individual-level reference groups in a SAR model allows the separate identification of social and contextual effects.<sup>21</sup>

Correlated effects are commonly addressed using fixed effects (Bifulco et al. 2011, Ajilore et al. 2014, Lin 2015, Horrace et al. 2016), and I follow this trend with the inclusion of classroom fixed effects. Intuitively, we can think of this as controlling for a teacher effect, although it also controls for other group treatments such as quality of the built environment, scheduled lunch time, and principle quality to name a few. Additionally, there are a number of other potential correlated effects. These can arise if certain types of students are differentially treated, either directly or through shared experience. For example, gender or racial groups may experience discrimination or differing levels of expectation and support. I include fixed effects for several groups, including racial, gender, free or reduced price qualification, and English language learner status.

The selection problem occurs when students select into peer groups with similar characteristics, such that outcomes may be correlated. In this paper, there are two levels of selection to consider: assignment into the classroom and into the friendship network. Selection into the classroom is the result of student residential location<sup>22</sup> and administrator decision. I test whether student characteristics explain classroom assignment and show that class assignment based on observed student characteristics is consistent with randomness in Appendix C.<sup>23</sup>

<sup>21.</sup> The intuition is that collinearity issues related to the characteristics and outcome of the reference group are broken up when individuals participate in more than one network, and these networks share some but not all members. To my knowledge, an early draft of Laschever (2013) was the first paper to propose this method. Bramoullé et al. (2009) formalize the partially overlapping reference group approach and shows explicit conditions for overcoming the reflection problem.

<sup>22.</sup> Exercising school choice is not uncommon in NYC public schools, but choice of school is based at least in part on residential location. Mader et al. (2018) describes the state of school choice in New York City. They find that the decision to opt into a choice school is heavily based on student residential location and the quality of their zoned school. I include residential zip code in an attempt to capture the similar choices faced by neighboring students. Explicitly modeling school choice is beyond the scope of this paper.

<sup>23.</sup> Of the tested variables, only English Language Learner (ELL) status appears not random. This is likely because public schools in New York state offer programs for ELL students, including transitional bilingual

Within the classroom, we could observe correlations between friends' test scores if students select friends based on academic performance, falsely identifying these correlations as peer effects. To address this, I use group fixed effects, which is a common method used to mitigate the problem of endogenous group formation.<sup>24</sup> The idea is to incorporate group effects for shared characteristics along which students may form friendships (homophily). These networks include gender and race, but in the baseline model, none of these directly represent academic performance. I address this with some alternate specifications in section 7 by binning student lag academic performance and controlling for within-performance selection. Results remain largely unchanged.

### 5.2 Baseline Model

This paper uses a revealed friendship network to measure academic spillovers in the classroom. For our baseline model, we construct a within-classroom network according to equation 4.1. In the linear in means model we use, this network is multiplied by both the outcome Yand student characteristics X so that we can separately identify endogenous and contextual effects. Below is the basic format of the linear in means model we estimate:

$$Y = \alpha + WY + WX + X\beta + \theta + U \tag{5.1}$$

where Y is the outcome of interest,  $\alpha$  is a constant, W is the weighting matrix as defined at the end of section 4.2,  $\theta$  is the classroom fixed effect,  $\beta$  is the estimate of own characteristics X, and U is the error term. Controls in X include lag test scores and indicators of sex, ethnicity, zip code, English language learning, and poverty status.

Modeling contextual effects allows us to control for the characteristics of students in the reference group, thereby isolating the endogenous effect of interest - the effect of one student's

and dual language programs.

<sup>24.</sup> This is particularly common in work using Add Health or other explicit networks, including Bramoullé et al. (2009), Lin (2010), Bifulco et al. (2011), Lin and Weinberg (2014), Hsieh and Lin (2017), and Patacchini et al. (2017).

performance on another's. The endogenous effect is important to distinguish from the contextual effect because it captures the spillover effects resulting from social interaction, whereas the contextual component controls for student characteristics. References to estimates of the peer effect refer to this endogenous effect.

I estimate this model using Maximum Likelihood Estimation (MLE) and follow Horrace et al. (2020) and Lee and Yu (2010). Details of the estimation procedure are found in Appendix A.

### 5.3 Interpretation

It is important to note that estimates of the endogenous effect from model 5.1 are multiplier effects. This means that interpretation of the estimated structural parameter  $\hat{\lambda}$  is done by converting the result as below:

$$\hat{\gamma} = \frac{1}{1 - \hat{\lambda}} \tag{5.2}$$

Thus an estimate of  $\hat{\lambda} = 0.05$  is interpreted as a multiplier of 1.053. This means that a ten percent improvement in test scores for a student's reference group results in a 0.53 percent improvement in the student's own test score. Notice that for small  $\lambda$ , the multiplier  $\gamma$  is comparable in magnitude.

### 6 Results

### 6.1 Baseline Results

Table 4 shows our baseline results for math and reading scores of fourth and fifth graders using a proximity measure in which students are next to one another in the lunch line. The outcome of interest is test scores, and these are z-scores normalized citywide among students in the same grade. In addition to the controls shown, the model also includes fixed effects for zip code of residence. We also include these zip codes in the contextual effect. The first line of Table 4 shows a math peer effect for students in the lunch line together of 0.078, which is statistically significant. As discussed in section 5.3 this is a multiplier effect, and so we interpret this as a multiplier of 1.085 or an increase of 0.085 units. The peer effect for reading is also significant, but smaller at 0.043, or a multiplier of 1.045. The fact that both estimates of the peer effect  $\lambda$  are positive is consistent with our intuition and the general findings of the literature, which is that improvements in the reference group should lead to improvements in own outcomes. We can interpret these results by saying that if a student's relevant peers exogenously improve their performance by one standard deviation, we expect to see improvements in own performance in math by 8.5% of a standard deviation. This is approximately 90% of the black-white test score gap in math. The gap is larger and the spillover effect is smaller in reading, so the equivalent improvement is equivalent in magnitude to about 35% of this gap.

It is difficult to associate meaning to a comparison of these estimates to static estimates because they are multipliers and therefore amplify all other elements of the education production function. We can think about the interpretation of these multiplier estimates when combined with additional external information and compute an average effect. The average classroom in our data has substantial variation in student ability, which we see manifest itself in student performance. If we collect the top performer in all classrooms, we find that the average classroom has a student performing 1.5 standard deviations above the mean.<sup>25</sup> This is mirrored in low performers.<sup>26</sup> I then collect the strongest connection we observe in each classroom, which we can think of as a student's best friend. The average student's largest connection is 0.357, meaning that over one third of the time we see both students present, they are next to one another in the lunch line. When we conduct our row normalization, the meaning is preserved, but the value of the matrix cell for the strongest connection reduces

<sup>25.</sup> In math, the average top performer across all classrooms scores 1.52 standard deviations better than the mean. For reading the average top performer scores 1.55 standard deviations above the mean.

<sup>26.</sup> The average bottom performer across all classrooms scores 1.39 standard deviations worse than the mean in math, and 1.51 standard deviations worse in reading

to 0.202. This means that the benefit to a student of connecting with the best student in the classroom, rather than an average student, is 0.026 in math and 0.014 in reading. This is equivalent to nearly half the effect of poverty and over one quarter of the black-white test score gap in math, and it stems from only one peer connection. In reading the effects are smaller than math, being one quarter the effect of poverty and 11% of the black-white test score gap (the gap is larger in reading).

The fact that the spillover is larger in math than reading is consistent with the idea that students learn verbal and reading skills at home, but primarily learn math in school. We see stronger in-school math effects than reading effects in Nye et al. (2004) which shows that teachers have a greater impact on math scores than reading scores.

Notice that the controls are performing as expected. Own student lag scores are highly significant and important. Male students perform slightly better in math but worse in reading than their female peers. English language learners and poor students do worse than native speakers and students who are not poor. The comparison group for ethnicity is Hispanic students, because these are the modal student in NYC public schools, and whites and Asians do better than them, while blacks do worse. Most of the contextual effects are not statistically important, with the exception of friends in the Asian/other group, which has a large positive impact. Taking the math estimate, this means having all friends in the Asian/other group improves own math performance by 0.18 standard deviations as opposed to having all Hispanic friends (the baseline reference group), all else equal. Notice that the contextual effect is not a multiplier effect, but simply shows the effect of having friends from this group type. Surprisingly, the previous performance of friends does not appear to matter in math, but it is quite important in reading, where gender of friends is also important.

### 6.2 Evolution of the friendship network over time

I construct the friendship network using repeated observations of the lunch line throughout the school year. This allows me to explore different periods within the school by defining separate networks for each period in the school year. I explore how the influence of the friendship network on test scores varies as the year progresses. It is ambiguous whether connections should be more important at the start or at the end of the year. Patacchini et al. (2017) provides evidence that students who are friends for longer periods of time are more important than short term friends, so we might expect that connections at the start of the year are more influential. On the other hand, students are still getting to know one another at the start of the year, so we may observe more noise as students sort into friendships. Additionally, testing occurs towards the end of the school year, so we might expect connections closer to the test date are most important.

Table 9 divides the year into between two and six periods in order to compare friendship importance over time. When dividing the year into halves, as in columns (1) and (6), I construct two proximity matrices according to equation 4.1. The first proximity matrix uses the set of days from the first half of the year, and the second matrix uses the days from the second half of the year. When dividing the year into additional periods, each matrix is constructed from the corresponding set of lunch line observations (days). This means that as we move between columns, the length of time represented by Period 1 is decreasing. In model (1), Period 1 represents about 90 school days, and in model (5) it represents about 30 school days.

Table 9 marks in bold the period in which the standardized test occurred. One common trend in all of these models is that the period after the test is unimportant. In models dividing the year into four or fewer periods (models 1-3 for math and 6-8 for reading) the first period appears important. This effect becomes insignificant when I include additional periods, but the test period remains significant throughout. Periods after the test never influence test scores. This supports my claim that these are peer effects, as we would not expect student behavior after a test to influence the test scores unless we were also picking up another signal.

### 7 Robustness Checks

#### 7.1 Random Lunch Lines

By construction, students in each of the networks we construct share a classroom, so we might expect that they are socially important to one another regardless of proximity in the lunch line. To test whether the spillover I estimate is simply a result of the students sharing a classroom, I randomly shuffle the lunch line for each day of the year and re-estimate the model. Results are found in Table 5. The placebo estimate for math is a statistically insignificant 0.004 and for reading it is also insignificant at 0.016. While the social effect (as well as all the components of the contextual effects) are insignificant, the own effects perform similarly to the baseline model. I conclude that students in close proximity to one another in the lunch line are socially more important to one another than a randomly selected classmate.

### 7.2 Alternate Distances in the Lunch Line

When constructing the baseline network, I modeled connections between students who are directly next to one another in line. If friendship groups are larger than pairs, a larger distance may be appropriate. For example, if we observe friends A, B, and C in line together, we miss the connection between A and C if we restrict our analysis to students who are next to one another. I explore alternate distances greater than one to test whether this is the appropriate group. Table 7 reports estimates when including multiple networks for each additional distance between two and six. While the inclusion of additional networks slightly dampens the effect from a distance of one, the results remain robust to the inclusion of these networks. Additionally, no higher distance is statistically significant. This indicates that students being next to one another in line is the strongest signal of connection we can measure. While students who are further apart in line may also be friends, this signal is too noisy to be meaningful.

It is possible to model higher thresholds within a single network (rather than putting each

distance into its own separate network). Table 8 shows the results of this network specification for the same set of distances. The effects appear to be stronger than our baseline model. Notice that point estimates for both math and reading increase and then decline. Standard errors are increasing for both outcomes, indicating additional noise from the inclusion of students who do not matter. These models are able to pick up larger friend groups, but they have higher potential to include non-friends as well. While Table 7 shows that no other distance is important on its own, these estimates pick up the extent to which larger friendship groups (or possibly friends of friends) are important.

#### 7.3 Removing potentially ordered classrooms

Some teachers may not give students agency over their position in line, assigning a specific order to their students. I introduced a measure M of within-classroom noise in section 4.1, which I discuss in greater detail in appendix B. M ranges between 0 under perfectly consistent ordering and 0.25 under uniform randomness. Figure 2 shows the distribution of M for all classrooms. The mean classroom has a noise measure of 0.203. Classrooms with low noise measure M indicate few changes in the ordering of students in the lunch line throughout the year. This could indicate external interference with a student's ability to choose their location in the lunch line. Appendix B further discusses properties and behaviors of the measure M.

Table 6 shows the results of removing classrooms which exhibit small levels of variation in the observed line order. I remove classrooms with M less than 0.05, 0.1, and 0.15. These values signify low levels of variation in lunch line order throughout the year, which may be attributable to students having no agency over their position in line. In each case, the peer effect increases in M, indicating that when students have the freedom to choose who they stand next to in the lunch line, the students they choose are more influential.

I provide this as evidence that the mechanism is friendship rather than the effect of time spent in the line. In classrooms with very stable line orders, students spend time in the lunch line with their neighbors more consistently than in classrooms with more variation. If the spillover mechanism was due to time spent together in the lunch line, we would expect a decrease in estimates when these classrooms are removed. We see the opposite, suggesting that connections we observe in classrooms with more autonomy are more meaningful. In addition to suggesting that friendship is the mechanism through which these spillovers are working, this suggests that peer effect estimates in my baseline model may be biased towards zero because of the additional noise added to the model by these ordered classrooms.

### 7.4 Alternate Proximity Matrix Specifications

There are other ways to define proximity in the lunch line. In this section I examine an alternative proximity definition to address a concern that low participation students may have outsized effects on those near them in the lunch line on the few days they participate. Section 4.2 describes our method for constructing the proximity matrices. Recall that the two main difficulties caused by student non-participation are a missed signal of who they choose to be near and removal of the absent student from the choice set of other students. The definition of proximity proposed here builds on this baseline measure. My previous definition of proximity measures the percent of days two students participate in lunch during which they choose to be near one another. To address the problem posed by low-participation students, I require students to participate a certain number of days together before their connection can be evaluated. The proximity matrix in equation (4.1) is altered such that:

$$p_{ij} = \begin{cases} \frac{\sum_{d=1}^{D} S_d(i,j)}{\sum_{d=1}^{D} \delta_d(i,j)}, & \text{if } i \neq j \text{ and } \sum_{d=1}^{D} \delta_d(i,j) \geq \eta \\ 0, & \text{otherwise} \end{cases}$$
(7.1)

where  $\eta$  is the threshold number of days students must both participate in before I count their connection. Connections between students when one or both of them are low participation students are reduced to zero unless we see enough participation from both students on the same days. A potential drawback of this method is the loss of all or most connections with low-participation classmates. However the signals we lose may be noisy.

Table 10 reports results for thresholds of between two and ten days that students must participate on the same day before we evaluate their connection. The results remain relatively unchanged in each specification.

### 7.5 Selection on Ability

The models presented so far include group fixed effects for a number of dimensions, including race, gender, and zip code. These are used to mitigate the problem of endogenous group formation.<sup>27</sup> We may be concerned that students select their friends based on ability as well. If this is the case, some correlation observed between the performance of friends would be due to the choice of students to be friends with classmates who perform similarly, rather than due to social spillovers. I replace the lag test score in my baseline model with a binned lag test score. We can think of lag test score as a measure of ability, and in this way I control for correlations between students with similar ability levels. I run multiple models in which I bin students using between two and ten quantiles, meaning these are groups of between 13 and 2.6 students for the average classroom.

Table 11 reports the results of this exercise. Estimates for the social effect remains large and significant. In math, the effect remains pretty similar, although it is dampened for the larger groups. For reading, the effect is consistently larger for all estimates. The pattern that emerges is that as group size decreases (more quantiles), the estimated social effect increases. This suggests that if fourth and fifth graders select their friends along ability lines, they do so at large levels of aggregation. That is, whether a classmate is in the same half or quartile of students may influence a student's selection of their friends, but students are less likely to care whether another student is at a specifically similar level.

27. See Bramoullé et al. (2009) for a discussion and application. Group fixed effects are common, including in Lin (2010), Bifulco et al. (2011), Lin and Weinberg (2014), Hsieh and Lin (2017), and Patacchini et al. (2017).

### 7.6 Friends from Last Year

I use classroom assignment to break up existing peer groups from the previous year, and have shown that classroom assignment is consistent with randomness on observed characteristics (Appendix C). But it could be that the connections I observe are simply friendships that carry over from the previous year, as some students in a current classroom surely shared the previous year classroom. I test whether this is a concerning source of selection. To do this, I measure the percentage of students shared a classroom the previous year. Precisely, the measure is:

$$S_{c} = \frac{1}{n_{c}} \sum_{i} \frac{1}{n_{c} - 1} \sum_{i \neq j} \mathbb{1}(i, j)$$
(7.2)

Where  $\mathbb{1}_d(i, j)$  is an indicator function that takes a value of one when *i* and *j* shared a classroom in the previous year, and zero otherwise.  $S_c$  is measured at the classroom (*c*) level, with each classroom having  $n_c$  students. This means that if  $S_c$  takes a value of 1, every student in the class shared a classroom the previous year, and a value of 0 means no students shared a classroom in the previous year.

I then split the sample by decile of  $S_c$  and estimate the social effect for math and reading for each subsample. If we see higher peer effect estimates for classrooms with higher values of  $S_c$ , this may indicate that friendships from the previous year carry over and are much more important than new current-year friendships. Figure 3 plots the results of these estimates for both math and reading test scores, with 95% confidence intervals. I find little to no pattern in these estimates, and the results remain relatively consistent across deciles.

### 7.7 Discussion

This section compares my results with other findings in the literature. It is important to note that estimates vary due to differences in context, methods, and definition of the reference group. The focus of this paper is the definition of the reference group, but to my knowledge there is no other work that explores an explicit friendship network for elementary school students.<sup>28</sup> This makes comparison challenging, as there will be differences in at least one of context or methods, as well as how the reference group is defined.

Lin (2010) uses the Add Health survey to estimate peer effects on GPA. She finds that an exogenous increase in peer performance by one standard deviation increases own performance by of 0.22 standard deviations. The methodology Lin uses is similar to that in this paper, but the context is quite different. Students in Add Health are middle and high school students (grades 7-12), so they are older than the elementary age students in this paper, and the scope of the network is school-level rather than within=classroom.<sup>29</sup>

Burke and Sass (2013) explore classroom effects for fourth and fifth graders in Florida public schools using a student fixed effects model. Similar to my results, they find larger effects in math (0.0292) than reading (0.0271), although the difference is less pronounced than in this paper. These effects are smaller than those presented here, but this could be due to the underweighting influential peer and overweighting unimportant peers that necessarily occurs when constructing reference groups at the classroom level.

It is possible that by narrowing in on the relevant peers exclusively addresses the concern of overweighting unimportant peers. Hong and Lee (2017) use assigned seats for Korean college students to show a hierarchy of which students are relevant. The pairs assigned to sit next to one another are most relevant, with influence decreasing for students sitting further away. They estimate peer effects between 0.02 and 0.03.<sup>30</sup> It is important to note the similarity in these estimates with my own discussion around the influence of a best friend. The key estimates in Hong and Lee (2017) involve only one other student in the classroom.

Lin and Weinberg (2014) also explores within-network heterogeneity in friendship effects,

<sup>28.</sup> Landini et al. (2016) is the closest I know, as they survey students in Italian primary schools to construct a friendship network. However they do not use this network to estimate peer effects, but rather explore differences in parent and student perceptions of school social networks.

<sup>29.</sup> Students in Add Health may select into similar classes, but this is not observed. Grades may be correlated with teachers (ex: a teacher may be more or less generous in their grading), and this is also not observed. These factors may lead to selection bias in these data that increases the correlation in academic outcomes between students. This issue may be less of a concern for behavior-based outcomes (such as smoking).

<sup>30.</sup> Specifically, they model the effect of their neighbor's midterm score on their final score in the class.

again using the Add Health data. They estimate effects for separate networks of friends in which connections are reciprocated, are nominated but not reciprocated, and are not chosen at all. As expected, the strongest effect is for students who reciprocate connections (0.17) followed by nominated friends (0.12), and finally peers that are not chosen (0.07). Notice that even the peers who are not nominated are found to be important for peer effects, although the effect is smaller.<sup>31</sup> This highlights the importance of not only exploring highly influential peers, but the network as a whole.

A single network that appropriately weights all classmates provides a view of the classroom peer effect as a whole. It takes into account the number of strong and weak connections for each student, such that we get a clear picture of the average peer effect in the classroom. There is benefit to separating the networks to understand the influencers, as well as to modeling the network as a whole. To my knowledge, this is the first paper to measure strength of connection between students and estimate the peer effect on academic outcomes.<sup>32</sup>

## 8 Conclusion

Models of peer effects and social networks typically define reference groups using group participation (ex: cohort or classroom) or a friendship network (ex: Add Health). These define the scope of the reference group, but without incorporating connection strength they necessarily underweight important peers and overweight important peers. In this paper, I construct a 'revealed' friendship network using repeated observations of which students choose to stand near one another in the lunch line.

I use this revealed friendship network to separately identify social and contextual effects and find evidence of significant social effects on both math and reading test scores. These effects are stronger in math than in reading. In math, a one standard deviation improvement

<sup>31.</sup> Their models include network fixed effects, which alleviates concern that we may simply be seeing withinschool correlations (ex: students in wealthier schools doing better). However, these estimates do not include a contextual effect, although they note that their results are robust to its inclusion.

<sup>32.</sup> De Giorgi et al. (2010) develops variation in connection strength using the number of shared classrooms college students attend. They estimate the peer effect on major choice.

in peer performance results in an increase in own performance between 7.5% and 11.1% of a standard deviation. This is a large effect, on par with my estimates of the performance gap between black and white students. Social spillovers have a multiplier effect, magnifying other inputs in the education production function. This suggests an alternative interpretation, which is that for a given intervention that improves math scores, between 7.5% and 11.1% of improvements occur through the peer effects mechanism. Spillovers are lower in reading, where an increase in peer performance of one standard deviation improves own performance by between 4.1% and 6.3% of a standard deviation. In addition, I exploit the daily nature of this data to look at the evolution of these connections over time and find evidence that social connections which occur during the test period are the most influential connections on test scores.

Many researchers and policymakers believe peer effects are consequential for a number of important policy discussions such as tracking, school choice, and school integration. In order to effectively weigh the costs and benefits of these policies, accurate estimates of social spillovers are imperative. Estimates that rely on the average peer effect within a reference group may understate the overall effect by not appropriately weighting the most relevant peers for each student, understating the influence of peers. Methods that measure the strength of social connections are critical for understanding how social networks operate.

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## 9 Tables

	NYC Studer	nt Population	Has POS	System		
	Freq.	Percent	Freq.	Percent	difference:	
Borough						
Manhattan	182,794	15.64%	164,258	15.59%	-0.06%	
Bronx	246,967	21.14%	218,101	20.70%	-0.44%	
Brooklyn	350,124	29.96%	$312,\!236$	29.63%	-0.33%	
Queens	$321,\!262$	27.49%	296,443	28.13%	0.64%	
Staten Island	$67,\!307$	5.76%	62,721	5.95%	0.19%	
Total:	1,168,454	100%	1,053,759	100%		
Ethnicity						
hispanic	472,229	49.76%	429,074	50.41%	0.66%	
black	302,744	31.90%	265,098	31.15%	-0.75%	
white	$174,\!105$	18.34%	156,966	18.44%	0.10%	
asian other	214,698	22.62%	198,241	23.29%	0.67%	
Total:	949,078	100%	851,138	100%		

Table 1: Comparing students in schools with a POS system to the full sample

Data are from the New York City Department of Education (NYCDOE). Table depict differences between all schools and those with a point of sale (POS) system for all students (over all grades). Ethnicity information is not known for all students.

Students	Number Drop	Percent Drop	Percent Remaining	Transactions	Number Drop	Percent Drop	Percent Remaining		
All 4th ar	All 4th and 5th graders at schools using POS systems for the full year								
145,495	0.0		100.00%	16,179,728	J J		100.00%		
Participat	te in school	l lunch		, ,					
140,090	$5,\!405$	3.71%	96.29%	$16,\!174,\!323$	$5,\!405$	0.03%	99.97%		
Have a te	st lag for e	ither mat	h or reading						
$129,\!234$	10,856	7.46%	88.82%	$15,\!118,\!176$	$1,\!056,\!147$	6.53%	93.44%		
Have a te	st score for	either ma	ath or reading						
$125,\!890$	$3,\!344$	2.30%	86.53%	$14,\!861,\!855$	$256,\!321$	1.58%	91.85%		
Transactio	on time is	between 1	0:00am and 2:0	0pm					
125,771	119	0.08%	86.44%	14,705,898	$155,\!957$	0.96%	90.89%		
Removing	; transactio	ons not oc	curing with stu	dent's class					
125,700	71	0.05%	86.39%	$14,\!582,\!405$	$123,\!493$	0.76%	90.13%		
Removing	; transactio	ons which	are simultaneo	us for the entire	class				
$125,\!559$	141	0.10%	86.30%	$14,\!568,\!447$	$13,\!958$	0.09%	90.04%		
Class size	is at least	20 studen	its						
$102,\!606$	$22,\!953$	15.78%	70.52%	$12,\!010,\!146$	$2,\!558,\!301$	15.81%	74.23%		

 Table 2: Sample Selection Process

The table depicts how many students (and corresponding transactions) are lost at each point of the sample selection process.

variable	mean	$\operatorname{sd}$	Ν
lunch_part_rate	0.658	0.275	102,606
lunch length	15.69	24.31	12,010,146
lunch time	12.07	0.79	12,010,146
class_size	25.57	3.21	$102,\!606$
female	0.506	0.500	$102,\!606$
grade4	0.489	0.500	102,606
grade5	0.511	0.500	$102,\!606$
ever poor	0.845	0.362	$102,\!606$
ell	0.125	0.331	$102,\!606$
ethnicity:			
hispanic	0.408	0.492	$102,\!606$
black	0.190	0.392	$102,\!606$
white	0.165	0.371	$102,\!606$
$asian_other$	0.237	0.425	$102,\!606$
Borough:			
manhattan	0.099	0.299	$102,\!606$
bronx	0.213	0.409	102,606
brooklyn	0.282	0.450	102,606
queens	0.335	0.472	$102,\!606$
$staten_{island}$	0.070	0.256	102,606
zmath	0.082	0.950	101,948
zread	0.088	0.953	102,244

Table 3: Summary Stats

Summary statistics for the selected sample. Lunch length calculated in minutes. Lunch time is in hours, so the mean lunch time is equivalent to 12:04.

	Ma	th	Read	ling
Social Effect:	0.078**	(0.009)	0.043**	(0.009)
Own Effect:		· · · ·		
lag test score	0.730**	(0.002)	0.660**	(0.003)
female	0.011**	(0.004)	-0.055**	(0.004
$\operatorname{ELL}$	-0.094**	(0.006)	-0.193**	(0.007)
Asian/other	0.177**	(0.005)	0.157**	(0.006
black	-0.019**	(0.005)	-0.043**	(0.006
white	$0.076^{**}$	(0.006)	0.088**	(0.007)
ever poor	$0.058^{**}$	(0.005)	$0.056^{**}$	(0.006
Contextual Effect:		· · · ·		<sup>×</sup>
lag test score	0.001	(0.010)	$0.056^{**}$	(0.011)
female	-0.006	(0.009)	0.030**	(0.011
$\operatorname{ELL}$	0.018	(0.021)	0.046	(0.027)
Asian/other	0.123**	(0.019)	$0.086^{**}$	(0.022)
black	-0.011	(0.020)	-0.038	(0.023
white	$0.046^{*}$	(0.021)	0.007	(0.025)
ever poor	0.003	(0.019)	-0.002	(0.023)
N	100, 156		94,838	

Table 4: Baseline Model

Models include classroom fixed effects, own zip code fixed effects, and zip code contextual fixed effects. Parameters with \* are significant at the 5% level and \*\* at the 1% level.

	Ma	Math		ling
Social Effect:	-0.016	(0.032)	-0.007	(0.033)
Own Effect:		· · · ·		,
lag test score	0.733**	(0.003)	$0.661^{**}$	(0.003)
female	-0.009**	(0.003)	0.053**	(0.004)
$\operatorname{ELL}$	-0.095**	(0.007)	-0.194**	(0.009)
Asian/other	0.180**	(0.006)	$0.159^{**}$	(0.007)
black	-0.021**	(0.006)	-0.048**	(0.007)
white	$0.077^{**}$	(0.007)	0.088**	(0.008)
ever poor	-0.060**	(0.006)	-0.056**	(0.007)
Contextual Effect:		· · · ·		· · · ·
lag test score	0.062	(0.039)	0.038	(0.042)
female	0.039	(0.043)	0.010	(0.051)
$\operatorname{ELL}$	-0.044	(0.083)	0.025	(0.105)
Asian/other	-0.047	(0.071)	-0.039	(0.082)
black	-0.061	(0.076)	-0.130	(0.088)
white	0.012	(0.077)	-0.018	(0.090)
ever poor	-0.028	(0.068)	0.024	(0.080)
N	100,156		94,838	

Table 5: Placebo

Models include classroom fixed effects, own zip code fixed effects, and zip code contextual fixed effects. Parameters with \* are significant at the 5% level and \*\* at the 1% level.

		Ma	ath	
	Baseline (1)	$\begin{array}{c} M \ge 0.05 \\ (2) \end{array}$	$\begin{array}{c} M \ge 0.1 \\ (3) \end{array}$	$\begin{array}{c} M \ge 0.15 \\ (4) \end{array}$
Social Effect	$\begin{array}{c} 0.078^{**} \\ (0.009) \end{array}$	$     0.083^{**} \\     (0.009) $	$ \begin{array}{c} 0.090^{**} \\ (0.010) \end{array} $	$0.093^{**} \\ (0.010)$
Ν	100,156	99,275	97,029	92,962
		Rea	ding	
	Baseline (5)	$\begin{array}{c} M \ge 0.05 \\ (6) \end{array}$	$\begin{array}{c} M \ge 0.1 \\ (7) \end{array}$	$\begin{array}{c} M \ge 0.15 \\ (8) \end{array}$
Social Effect	$\begin{array}{c} 0.043^{**} \\ (0.009) \end{array}$	$ \begin{array}{c} 0.048^{**} \\ (0.010) \end{array} $	$0.052^{**} \\ (0.010)$	$0.056^{**} \\ (0.011)$
N	94,838	94,022	91,920	88,121

Table 6: Removing classrooms with little variance in observed line order

Models include classroom fixed effects, own zip code fixed effects, and zip code contextual fixed effects. Parameters with \* are significant at the 5% level and \*\* at the 1% level. Columns (1) and (5) are reproduced from Table 4. Other columns present results from the same model run on subsamples excluding classrooms with a measure of within-classroom noise (M) less than the specified threshold.

		Ma	ath					
Social Effect:	(1)	(2)	(3)	(4)				
D=1	0.078**	0.082**	0.082**	0.081**				
	(0.009)	(0.009)	(0.009)	(0.009)				
D=2		0.005	0.004	0.004				
		(0.012)	(0.012)	(0.012)				
D=3			0.007	0.011				
			(0.013)	(0.013)				
D=4				-0.024				
				(0.013)				
N	100,156	100,156	100,156	100,156				
	Reading							
Social Effect:	(5)	(6)	(7)	(8)				
D=1	0.043**	0.045**	0.044**	0.043**				
	(0.009)	(0.010)	(0.010)	(0.010)				
D=2	( )	0.011	0.015	0.014				
		(0.012)	(0.013)	(0.013)				
D=3			-0.018	-0.018				
			(0.013)	(0.013)				
D=4				-0.015				
				(0.014)				

 Table 7: Multiple Distance levels

Models include classroom fixed effects, own zip code fixed effects, and zip code contextual fixed effects. Parameters with \* are significant at the 5% level and \*\* at the 1% level. Columns (1) and (5) are reproduced from Table 4, which defines proximity between students as students standing directly next to one another in the lunch line on a given day. Other columns present results from a similar model that differs only with the addition of networks for distances greater than one. For example, columns (2) and (6) adds a network for students standing in the lunch line with a single student between them. The inclusion of these additional networks coincides with the inclusion of contextual effects for each network.

94,838

94,838

94,838

94,838

N

		М	ath						
	D = 1 (1)	$\begin{array}{c} \mathrm{D} \leq 2\\ (2) \end{array}$	$\begin{array}{c} \mathrm{D} \leq 3\\ (3) \end{array}$	$\begin{array}{c} \mathrm{D} \leq 4\\ (4) \end{array}$					
Social Effect	$\begin{array}{c} 0.078^{**} \\ (0.009) \end{array}$	$     0.099^{**} \\     (0.013) $	$ \begin{array}{c} 0.114^{**} \\ (0.016) \end{array} $	$\begin{array}{c} 0.112^{**} \\ (0.019) \end{array}$					
N	100,156	100,156	100,156	100,156					
	Reading								
	D = 1 (5)	$\begin{array}{c} \mathrm{D} \leq 2\\ (6) \end{array}$	$\begin{array}{c} \mathrm{D} \leq 3\\ (7)\end{array}$	$\begin{array}{c} \mathrm{D} \leq 4\\ (8)\end{array}$					
Social Effect	$\begin{array}{c} 0.043^{**} \\ (0.009) \end{array}$	$     0.064^{**} \\     (0.013) $	$0.062^{**} \\ (0.017)$	$\begin{array}{c} 0.054^{**} \\ (0.020) \end{array}$					
N	94,838	94,838	94,838	94,838					

Table 8: Distances greater than one

Models include classroom fixed effects, own zip code fixed effects, and zip code contextual fixed effects. Parameters with \* are significant at the 5% level and \*\* at the 1% level. Columns (1) and (5) are reproduced from Table 4, which defines proximity between students as students standing directly next to one another in the lunch line on a given day. Other columns present results from a similar model that differs only with the increase of the bandwidth to distances greater than one. For example, columns (2) and (6) considers students friends if they are directly next to one another in the line, or have one student between them.

			Math		
Social Effect:	(1)	(2)	(3)	(4)	(5)
Period 1	0.040**	0.027**	0.020*	0.014	0.007
	(0.010)	(0.010)	(0.009)	(0.009)	(0.009)
Period 2	0.040**	0.032**	$0.022^{*}$	0.007	0.016
	(0.010)	(0.011)	(0.010)	(0.010)	(0.009)
Period 3		0.021*	0.028**	0.032**	0.017
		(0.010)	(0.010)	(0.010)	(0.010)
Period 4			0.006	0.030**	0.011
			(0.010)	(0.010)	(0.010)
Period 5				-0.009	$0.030^{**}$
				(0.009)	(0.010)
Period 6					-0.009
					(0.009)
N	100,156	100,156	100,156	100,156	100,156
			Reading		
Social Effect:	(6)	(7)	(8)	(9)	(10)
Period 1	0.030**	0.029**	0.021*	0.014	0.010
	(0.011)	(0.010)	(0.010)	(0.009)	(0.009)
Period 2	0.019	0.002	-0.001	0.012	0.015
	(0.010)	(0.011)	(0.010)	(0.010)	(0.010)
Period 3		0.020*	0.024*	-0.001	-0.008
		(0.010)	(0.011)	(0.010)	(0.010)
Period 4			0.002	$0.027^{**}$	0.007
			(0.010)	(0.010)	(0.010)
Period 5				-0.003	$0.021^{*}$
				(0.009)	(0.010)
Period 6					0.001
					(0.009)
N	94,838	94,838	94,838	94,838	94,838

Table 9: Evolution of friendship importance over the school year

Each column presents a model in which I divide the school year into additional periods, such that friends from each period are considered separate networks. Periods in bold denote the period in which the standardized test occured. Models include classroom fixed effects, own zip code fixed effects, and zip code contextual fixed effects. Parameters with \* are significant at the 5% level and \*\* at the 1% level.

	Math								
	$\begin{array}{c} \text{Days} \ge 1\\ (1) \end{array}$	$\begin{array}{c} \text{Days} \geq 2\\ (2) \end{array}$	$\begin{array}{c} \text{Days} \geq 3\\ (3) \end{array}$	$\begin{array}{c} \text{Days} \ge 4\\ (4) \end{array}$	$\begin{array}{c} \text{Days} \geq 5\\ (5) \end{array}$	$\begin{array}{l} \text{Days} \ge 6\\ (6) \end{array}$			
Social Effect	$\frac{0.078^{**}}{(0.009)}$	$     0.079^{**} \\     (0.009) $	$     0.079^{**} \\     (0.009) $	$     0.076^{**} \\     (0.009) $	$ \begin{array}{c} 0.075^{**} \\ (0.009) \end{array} $	$\begin{array}{c} 0.072^{**} \\ (0.009) \end{array}$			
N	100,156	100,156	100,156	100,156	100,156	100,156			
			Rea	ding					
	$\begin{array}{c} \text{Days} \ge 1\\ (7) \end{array}$	$\begin{array}{c} \text{Days} \geq 2\\ (8) \end{array}$	$\begin{array}{c} \text{Days} \geq 3\\ (9) \end{array}$	$\begin{array}{l}\text{Days} \ge 4\\(10)\end{array}$	$\begin{array}{c} \text{Days} \ge 5\\ (11) \end{array}$	$\begin{array}{l} \text{Days} \ge 6\\ (12) \end{array}$			
Social Effect	$\frac{0.043^{**}}{(0.009)}$	$     0.044^{**} \\     (0.010) $	$     0.044^{**} \\     (0.010) $	$     0.043^{**} \\     (0.010) $	$     0.042^{**} \\     (0.010) $	$\begin{array}{c} 0.042^{**} \\ (0.010) \end{array}$			
Ν	94,838	94,838	94,838	94,838	94,838	94,838			

Table 10: Students must participate together more than one day

Columns (1) and (5) are reproduced from Table 4, which has no minimum requirement for number of days students must stand next to one another in order to consider a connection between them. Other columns present results from a similar model that differ only with the introduction of a minimum number of days. For example, the models represented in columns (2) and (6) eliminate a connection between two students unless they are next to one another in the lunch line for at least two days. This minimum number of days is increased to three in columns (3) and (7), and to four in (4) and (8). Models include classroom fixed effects, own zip code fixed effects, and zip code contextual fixed effects. Parameters with \* are significant at the 5% level and \*\* at the 1% level.

				IVI	INTAULT			
	Baseline (1)	2 Qtiles (2)	3 Qtiles (3)	4 Qtiles (4)	5 Qtiles (5)	6 Qtiles (6)	8 Qtiles (7)	10 Qtiles (8)
Social Effect	$\begin{array}{c} \text{Social} & 0.078^{**} \\ \text{Effect} & (0.009) \end{array}$	$0.053^{**}$ (0.009)	$0.053^{**}$ (0.009)	$0.049^{**}$ (0.009)	$0.057^{**}$ (0.009)	$0.078^{**}$	$0.082^{**}$ (0.009)	$0.093^{**}$ (0.09)
	100, 156	100, 156	100, 156	100, 156	100, 156	100, 156	100, 156	100,156
				Rea	Reading			
	Baseline (9)	2 Qtiles (10)	3 Qtiles (11)	4 Qtiles (12)	5 Qtiles (13)	6 Qtiles (14)	8 Qtiles (15)	$\begin{array}{c} 10 \text{ Qtiles} \\ (16) \end{array}$
Social Effect	$\begin{array}{c} \text{Social} & 0.043^{**} \\ \text{Effect} & (0.009) \end{array}$	$0.075^{**}$ (0.09)	$0.061^{**}$ (0.009)	$0.075^{**}$ (0.009)	(000.0)	$0.087^{**}$ (0.009)	0.104 (0.069)	$0.127^{**}$ (0.009)
	94,838	94,838	94,838	94,838	94,838	94,838	94,838	94,838

Table 11: Binned Lag Test Scores

The number of quantiles increases to three in columns (3) and (11), to four in columns (4) and (12), and so on. Models include classroom fixed effects, own zip code fixed effects, and zip code contextual fixed effects. Parameters with \* are significant at the 2000 od Spr cor nne orr 11 17 and (TA) monage 5% level and \*\* at the 1% level. in columns (2) Columns similar m

## 10 Figures

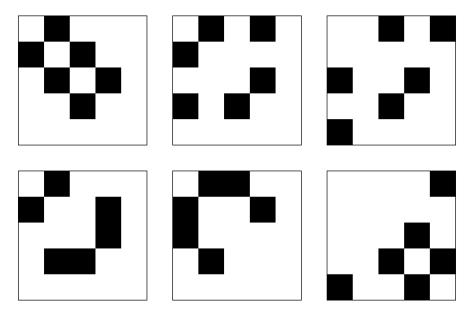
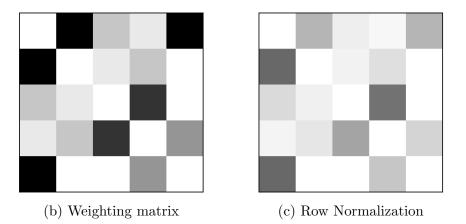


Figure 1: Constructing the Weighting Matrix

(a) Six example daily lunch line observations



Example daily observations in (a) are constructed from example line orders [A,B,C,D]. [C,D,A,B], [D,C,A,E], [C,D,B,A], [D,B,A,C], and [A,E,D,C]. (b) Shows the resulting weighting matrix, constructed using equation 4.1 and (c) shows the result of row-normalization.

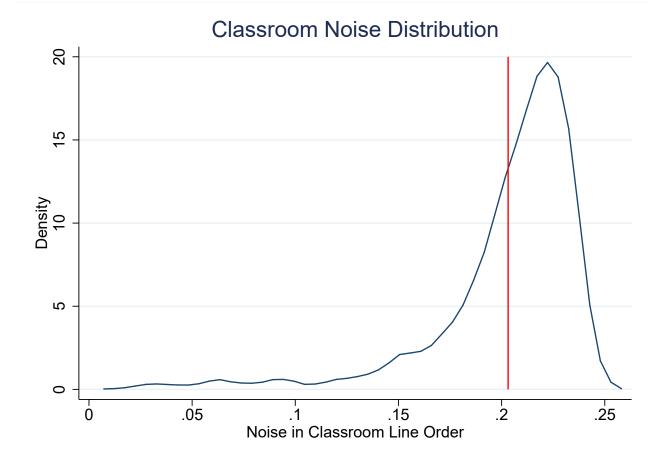


Figure 2: Classroom Noise Distribution

Vertical line indicates mean, which is equal to 0.203. Includes one entry for each of 4,077 classrooms. Density constructed using an Epanechnikov kernel with bandwidth = 0.004.

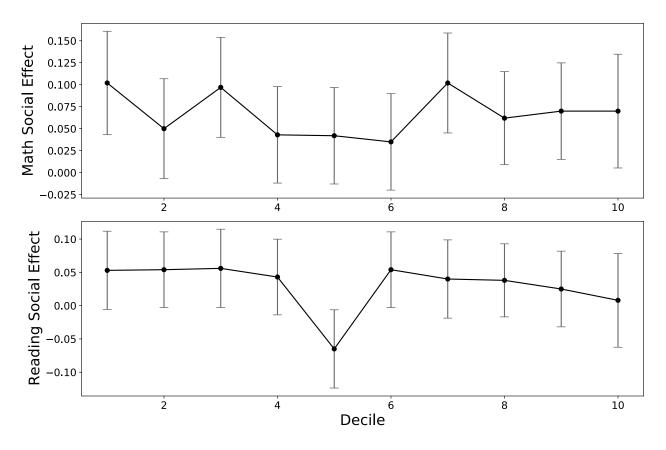


Figure 3: Classroom Noise Distribution

Graph reports estimates from ten models for each outcome. Models are subsamples for each decile of measure  $S_c$ , a measure of how many students in the classroom were classmates the year prior. Bars indicate a 95% confidence interval.

#### A Estimation procedures

I estimate a spatial autoregressive (SAR) model of the form:

$$Y = \lambda WY + \theta WX + X\beta + U \tag{A.1}$$

Where X contains a constant and the fixed effects for simplicity of notation. I estimate the model using maximum likelihood estimation (MLE), and so assume U is  $iid(0, \sigma^2)$ . Note that if we do not assume normality of the error term, this becomes quasi-maximum likelihood estimation (QMLE).

In order to maintain the interdependencies of the error terms and incorporate classroom fixed effects, I follow the transformation approach discussed in Lee and Yu (2010) and used in Horrace et al. (2020). This method involves the deviation from the classroom mean operator  $Q = \iota l'/n$  an  $n \times n$  matrix where n is classroom size. I define the orthonormal within transformation matrix Q as  $[P, \iota_n/\sqrt{n}]$ . Following Lee and Yu (2010), I premultiply our model by P':

$$P'Y = \lambda P'WY + P'\theta WX + P'X\beta + P'U \tag{A.2}$$

This means our log likelihood function takes the form:

$$ln\mathcal{L}(\lambda,\beta,\sigma^2) = -\left(\frac{n-1}{2}\right) \left[ ln(2\pi) + ln(\sigma^2) \right] + ln|I - \lambda P'WP| - \frac{\bar{e}'Q\bar{e}}{2\sigma^2}$$
(A.3)

Where  $\bar{e} = P'Y - \lambda P'WQY - P'WQX\theta - P'X\beta$  After some algebra, we can rewrite this with only Q (and not P):

$$ln\mathcal{L}(\lambda,\beta,\sigma^2) = -\left(\frac{n-1}{2}\right) \left[ ln(2\pi) + ln(\sigma^2) \right] - ln(1-\lambda) + ln|I-\lambda W| - \frac{e(\lambda,\xi)'Qe(\lambda,\xi)}{2\sigma^2}$$
(A.4)

where  $\xi = (\theta, \beta)', \ \mu = (WX, X)$  and  $e(\xi) = Y - \lambda WY - WX\theta - X\beta = Y - \lambda WY - \mu\xi.$ 

Notice that the parameter space for  $\lambda$  must be restricted such that its magnitude is less than one in order to guarantee that both  $|I - \lambda W|$  will be strictly positive and  $\ln(1 - \lambda)$  is well defined.

I simplify estimation by concentrating out the  $\xi$  and  $\sigma^2$  using first order conditions. Thus:

$$\xi^{\star}(\lambda) = (\mu' Q \mu)^{-1} \mu' Q (Y - \lambda W Y) \tag{A.5}$$

and

$$\sigma^{2\star}(\lambda,\xi) = \frac{e'(\lambda,\xi)Qe(\lambda,\xi)}{n-1} \tag{A.6}$$

This simplifies estimation substantially, as we need only maximize in one dimension. We get some cancellation from the  $\sigma^{2*}$  and our likelihood function for an individual class becomes:

$$ln\mathcal{L}(\lambda) = -\left(\frac{n-1}{2}\right) \left[ ln(2\pi) + 1 + ln[\sigma^{2\star}(\lambda)] \right] - ln(1-\lambda) + ln|I - \lambda W|$$
(A.7)

I sum the likelihoods over all classrooms to obtain the complete likelihood, analogous to the way Lee and Yu (2010) sum over the time periods.

In order to calculate the standard errors, I again follow Lee and Yu (2010) and estimate the asymptotic variance matrix  $V_{ML}$  as in the block matrix below:

$$V_{ML} = \begin{pmatrix} a & d & e \\ d & b & 0 \\ e & 0 & c \end{pmatrix}^{-1}$$
(A.8)

Such that:

$$a = \frac{\partial \ln \mathcal{L}(\lambda, \xi, \sigma^2)}{\partial \lambda_k \partial \lambda_l} = (W_k G)' Q W_l G / \sigma^2 + \operatorname{tr}[W_k G Q W_l G]$$

$$a = \frac{\partial \ln \mathcal{L}(\lambda, \xi, \sigma^2)}{\partial \xi^2} = \mu' Q \mu / \sigma^2$$

$$c = \frac{\partial \ln \mathcal{L}(\lambda, \xi, \sigma^2)}{\partial \sigma^4} = (n-1)/(2\sigma^4)$$

$$d = \frac{\partial \ln \mathcal{L}(\lambda, \xi, \sigma^2)}{\partial \lambda_k \partial \xi} = (W_k G)' Q \mu \sigma^2$$

$$e = \frac{\partial \ln \mathcal{L}(\lambda, \xi, \sigma^2)}{\partial \lambda_k \partial \sigma^2} = \operatorname{tr}[Q W_k G] / \sigma^2$$
(A.9)

where  $G = (I - \sum_k \lambda_k W_k)^{-1}$ . The standard errors are then the square roots of the diagonal of  $V_{ML}$ .

#### **B** Measuring Order Noise

When analyzing a social network based off the observed lunch line order, we may be concerned that students do not have agency over their place in line. If the students are ordered by some external factor, such as a teacher, then the interpretation of our results changes. As such, I attempt to determine whether there is a large set of classrooms in which students are ordered. The measure I intend to create will be able to determine whether a consistent order is used throughout the period of observation. If a teacher orders students alphabetically (for example) for lunch every day, I will detect this line order as having little noise.

In determining a good measure of noise, the measure must have two specific characteristics. First, the measure needs to be invariant to classroom size so that we can compare noise across classrooms without concern that the driving factor is number of students. The second issue is that students do not participate every day, so the measure must be able to contend with varying student combinations and line sizes. Thus any bias in our measure cannot be a function of class size or lunch participation rate. For the purpose of developing intuition, I discuss first an intuitive measure that does not meet these criteria, and then its relationship to a measure that does.

Consider every pair of students (i, j) within a classroom. For these students, I define two quantities  $A_{ij}$  and  $B_{ij}$ . Let r(i) be the rank of student i and  $\mathbb{1}_d(i, j)$  be an indicator function for both students i and j being present at lunch on day d. Then  $A_{ij} = \sum_D \mathbb{1}_d (r(i) < r(j))$ and  $B_{ij} = \mathbb{1}_d(i, j)$ . These quantities allow us to determine the number of switches  $C_{ij} =$ min  $(A_{ij}, B_{ij} - A_{ij})$  if we assume that the most common order is the "true" order of the students. The quantity  $S = \sum_{i < j} C_{ij}$  gives the total number of inversions, and I normalize this by the number of observed pairs  $B = \sum_{i < j} B_{ij}$ , so that our noise measure is M = S/B. This has a nice interpretation as the chance that a given pair is swapped. However the measure does not quite have the properties we would like - the measure varies by size and participation rate.

To understand the issue, I look at the bias of our measure. For all pairs (i, j), there exists

some probability q of swapped order, conditioning on the appearance of both students (i, j). This results in the expectation of the total number of times i and j swap order equal to  $q \cdot B_{ij}$ . I consider the expectation of our estimator:  $\mathbb{E}[C_{ij}] = \sum_{k=0}^{B_{ij}} {B_{ij} \choose k} q^k (1-q)^{(B_{ij}-k)} \min(k, B_{ij} - k)$ . This expectation varies with  $B_{ij}$ . When  $B_{ij} \in \{0, 1\}$ ,  $\mathbb{E}[C_{ij}] = 0$ ; when  $B_{ij} \in \{2, 3\}$ ,  $\mathbb{E}[C_{ij}] = B_{ij}q(1-q)$ ; and more complex objects as  $B_{ij}$  increases. I look for a  $\tilde{C}_{ij}$  where  $\mathbb{E}[\tilde{C}_{ij}] = B_{ij}q(1-q)$  regardless of  $B_{ij}$  (being invariant in  $B_{ij}$  should meet the requirements of invariance to class size and participation rate). To do this, I replace  $\min(k, B_{ij} - k)$  with  ${B_{ij}^{-1}}^{-1}{B_{ij}^{-2}} = \frac{k(B_{ij}-k)}{B_{ij}-1} = \varphi$ . What is nice about  $\varphi$  is that it keeps much of the meaning of  $\min(k, B_{ij} - k)$ . Without loss of generality, we can say that  $\min(k, B_{ij} - k) = k$ . Notice that in both measures,  $C_{ij=0}$  when k = 0, so I consider only k > 0. Then  $1 \le k \le \frac{B_{ij}}{2}$ . Thus  $\frac{B_{ij}}{2} \le B_{ij} - k \le B_{ij} - 1$ . This implies  $\frac{k}{2} < \frac{kB_{ij}}{2(B_{ij}-1)} \le \frac{k(B_{ij}-k)}{B_{ij}-1} = \varphi \le k$ , and we see that  $\varphi$ is bounded by k and  $\frac{k}{2}$ , although it loses the nice interpretation of our estimator being the chance i and j are swapped. We do however gain invariance by size and absences, which I will show when we finish constructing the measure. As before, I normalize this by dividing by the number of observed pairs. The result is:

$$M = \frac{\sum_{i < j; B_{ij} > 1} \tilde{C}_{ij}}{\sum_{i < j; B_{ij} > 1} B_{ij}} \quad \text{where} \quad \tilde{C}_{ij} = \frac{A_{ij}(B_{ij} - A_{ij})}{B_{ij} - 1}$$
(B.1)

The omission we are left with is the case for  $B_{ij} = 1$ . Given probability p that a pair of students participate in lunch, the expectation that the students participate in lunch together only one time is  $\mathbb{E}[B_{ij} = 1] = Dp(1-p)^{(D-1)}$ . Average lunch participation is 65.8% over a school year of 180 days.<sup>33</sup> If two students participated 25% of the time (such that p = .0625), the chance of  $B_{ij} = 1$  is less than  $e^{-9}$  if the participation of students i and j is independent. Given such a small chance, I ignore the scenario  $B_{ij} = 1$  and forcibly remove such occurrences from the measure.

We can see that our measure is invariant to class size and participation rate in Table B1,

<sup>33.</sup> The school year is required to be at least 180 days, and is sometimes longer than this. In 2013, the school year was exactly 180 days.

which reports results of Monte-Carlo simulations under changes in class size and participation rate. Line orders are generated randomly for each of 180 days to simulate observation throughout the school year. Standard errors decrease in participation rate.

The measure is meant to detect noise, so I also simulate increases in randomness to show that the measure works as promised. Table B2 reports results of Monte-Carlo simulations on the measure of classroom noise in response to increasing levels of randomness. Line orders are generated randomly for X% of the 180 days, where X is in the percent random column. In all of these simulations, students have a 70% chance of participation in the line on any simulated day. The measure increases as randomness increases. I also show what may be apparent from the previous discussion, which is that when students are perfectly ordered in the classroom, the measure is zero. The average measure observed in the data is 0.203, which is consistent with between 50% and 60% randomness in the lunch line. This makes sense, as we expect variation in the line order, but as students reveal their preferences for line location and who to be in line with, we expect the sorting to be less than random. It is likely that classroom geography also plays a part in which groups of students are most likely in the front of the line on a consistent basis, further reducing the number of inversions detected by the measure. We expect that the lower level of randomness is not restricted to specific days of the year, as in the simulations, but rather each day has non-random variation.

It is important to note that this measure will be limited if the teacher attempts to be more equitable and alternates (for example) lining their students up alphabetically one day and reverse alphabetically another day, I would not detect this as an ordering (because there will be many line switches even without large changes in relative position). While limited in this way, I argue that the majority of orderings we might be concerned with (ex: alphabetical, height, location with the classroom, etc) will be detected by this measure.

### C Classroom Assignment

There are two forms of selection that are important to address. The first is the assignment of students to their set of potential peers. This occurs through two channels: assignment of students to school and then to classrooms. The second is the selection of friends within the classroom.<sup>34</sup> In this section I provide evidence that, conditional on school attended, student assignment into classrooms is consistent with randomness over most observed characteristics.

A key assumption for our estimates to be causal is that that assignment of students to their choice of peers (classroom assignment) is random. I have no insight into the assignment process, but I do show that over most observed characteristics, assignment is consistent with randomness. The objective of this test is to show that classroom assignment is not a function of the observed characteristics along which sorting into friendships might occur. To do this, I consider a series of multinomial logits as follows:

$$Class_i = \alpha + X_i \beta_{qst} + \varepsilon_i \tag{C.1}$$

For each iteration of equation (C.1) I include a single grade g within a single school s, during a single year t. I exclude all school-grades for which there is only a single classroom, as these schools by definition assign their students to classrooms randomly (less than 5% of our sample are in cohorts with only one classroom).  $Class_i$  indicates the classroom assignment for student i, and the number of options varies by school-grade.<sup>35</sup>  $X_i$  is a binary indicator variable for a characteristic of student i. Each iteration of equation (C.1) gives us an estimate  $\beta_{gst}$  and a t-statistic. The t-statistic tells us the significance of the characteristic for assignment at that

<sup>34.</sup> In the current version, this within-classroom friendship selection is dealt with primarily through group fixed effects. Homophily plays an important role in who students select as their friends, and the models used in this paper include a large set of demographic characteristics to control for these sorting avenues - including gender, ethnicity, residential zip code, and others. This is in line with other literature which uses fixed effects for networks of importance to control for these sorting effects. However, I include additional information in my measure of connection strength. To the extent that this additional information is the result of sorting which is not controlled for by these avenues, further work needs to be done. Future versions of this paper will include a more thorough examination of this within-classroom sorting.

<sup>35.</sup> There are between 2 and 11 classrooms in a school-grade-year. The mean is 4.2 classrooms.

school-grade, and I collect the t-statistic for all school-grades. I then conduct my own random assignment of students to classrooms and run the same set of models, again collecting these t-statistics. I then compare the distributions of t-statistics from the observed and simulated models.

Figures A1 and A2 show the results of these tests. Each dot in these figures compares equal ranked t-statistics from the simulated and observed populations. If these distributions are the same, we should expect a 45 degree line. Most of the observed characteristics remain reasonably close to the 45 degree line, with the largest deviation at the tails of the distribution. A notable exception is the English language learner characteristic, which appears to deviate significantly from the 45 degree line. This indicates that classroom assignment may group English language learners into classrooms together. It is important to note that this test behaves best when the group sizes are similar in size, such as when we compare female and male students. English language learners make up only 12.5% of the population. That said, this is similar (slightly smaller than) the size of both white and Asian/other students in our sample, and both of these groups appear to behave better in this visual test. Thus, with the exception of English language learners, I conclude that we do not need to be concerned about selection into classrooms based on observed characteristics.

# **D** Appendix Tables and Figures

Participation rate:	Class size 20		Clas	s size 25	Class	size 30
25%	0.2501	(0.0027)	0.25	(0.0023)	0.2501	(0.0019)
50%	0.25	(0.0008)	0.25	(0.0007)	0.25	(0.0006)
75%	0.25	(0.0005)	0.25	(0.0004)	0.25	(0.0004)
100%	0.25	(0.0003)	0.25	(0.0003)	0.25	(0.0003)

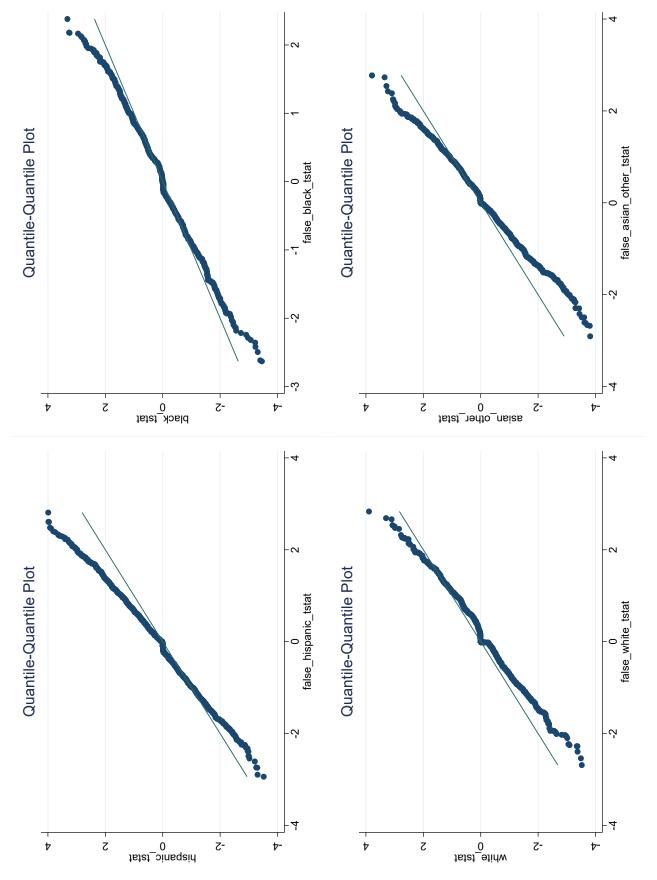
Table B1: Monte Carlo simulations of classroom size and participation rates

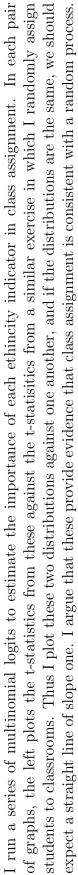
Table reports the results of Monte-Carlo simulations on the measure of classroom noise. Simulation is for 1,000 classrooms at each combination of class size and participation rate. Standard errors are in parenthesis. Line orders are generated randomly for each of 180 days to simulate observation throughout the school year. The measure is invariant to class size and participation rate, although standard errors increase as participation rate decreases.

Percent random:	mean	s.e.
0%	0.0000	(0.0000)
10%	0.0489	(0.0033)
20%	0.0902	(0.0040)
30%	0.1287	(0.0042)
40%	0.1603	(0.0039)
50%	0.1886	(0.0034)
60%	0.2104	(0.0029)
70%	0.2282	(0.0022)
80%	0.2402	(0.0015)
90%	0.2478	(0.0008)
100%	0.2500	(0.0004)
Ν	10,000	

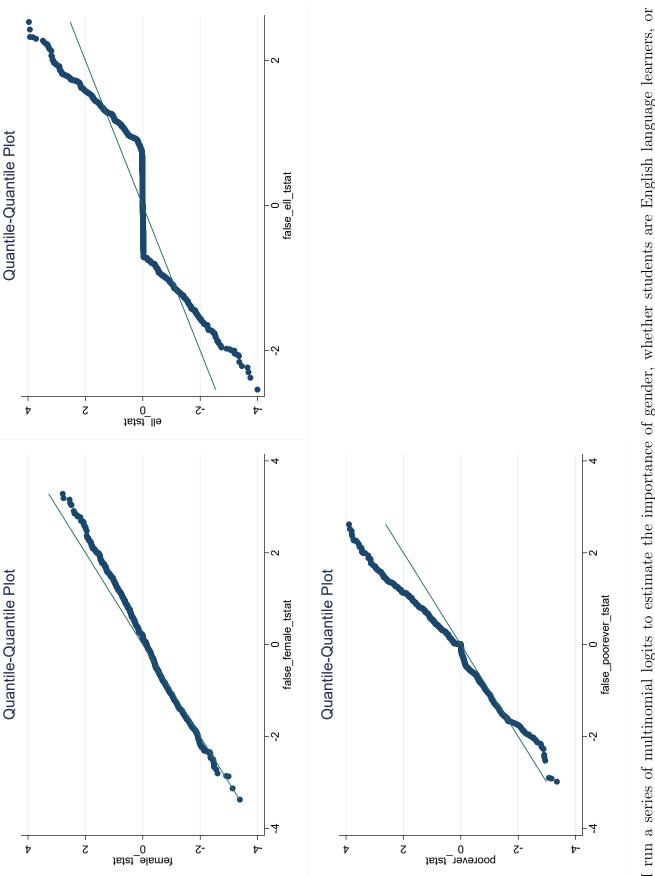
Table B2: Monte Carlo simulations for dif-ferent levels of randomness

Table reports results of Monte-Carlo simulations on the measure of classroom noise. Simulation is for 10,000 classrooms at each combination of class size and participation rate. Line orders are generated randomly for X% of the 180 days, where X is in the percent random column. Students have a 70% chance of participation in the line on any simulated day. The measure increases as randomness increases.









whether the student rides is poor. In each pair of graphs, the left plots the t-statistics from these against the t-statisitics from a I run a series of multinomial logits to estimate the importance of gender, whether students are English language learners, or if the distributions are the same, we should expect a straight line of slope one. I argue that these provide evidence that class assignment is consistent with a random process for most observed characteristics. This test performs best when groups are large (ex: females are about half the student population) and is more noisy when the network is small (ex: English language learners similar exercise in which I randomly assign students to classrooms. Thus I plot these two distributions against one another, and are only about 12.5% of the student population).